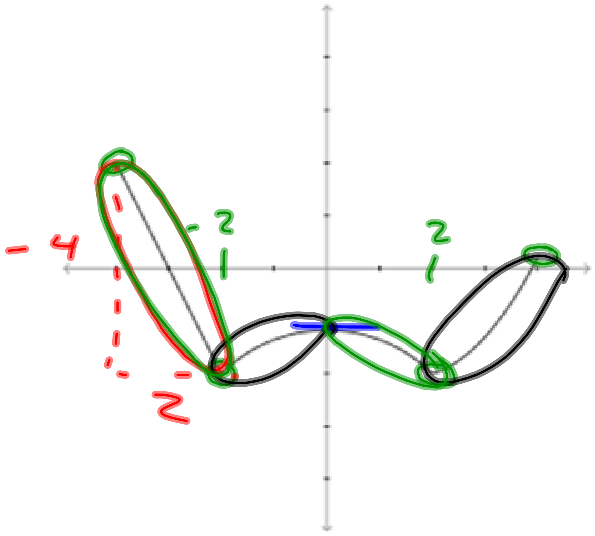
1/9/17 "Do what you are supposed to do."-Mr. Callahan

HW: "Graphs of Functions and Derivatives HW"
Test 3 on Thursday 1/12
AIM: Graphing Functions and Derivatives
Warm Up:
1) What does the 1st Derivative tell us?

2) What does the 2nd Derivative tell us?

Consider the graph of $f(x)$ below:



1. Use the graph to answer the following questions.

   (a) Are there any values x for which the derivative $f'(x)$ does *not* exist?

   $x = \pm 2$ $x = \pm 4$ corners, discontinuities

   (b) Are there any values x for which $f'(x) = 0$?

   $x = 0$ (horizontal tangent) (Max/Min)

(c) This particular function f has an interval on which its derivative $f'(x)$ is constant. What is this interval? What does the derivative function look like there? Estimate the slope of $f(x)$ on that interval.

$(-4,-2)$

horizontal line

Slope estimate: $-\frac{4}{2} = -2$

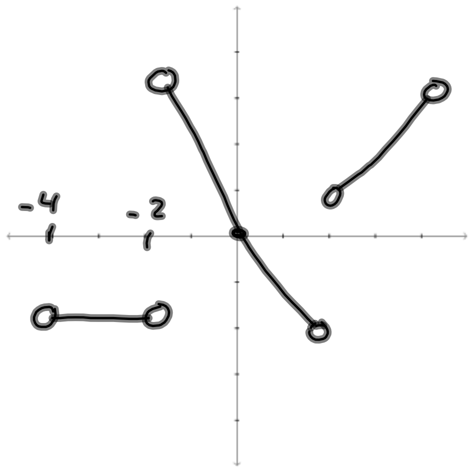
(d) On which interval or intervals is $f'(x)$ positive?

f(x) is increasing $(-2,0) \cup (2,4)$

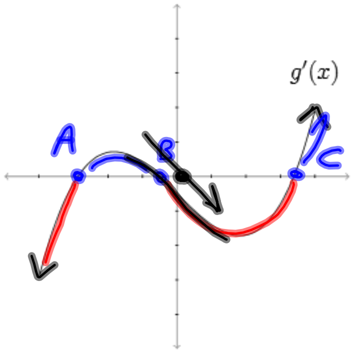
(e) On which interval or intervals is $f'(x)$ negative? Again, sketch a graph of the derivative on those intervals.

f(x) is decreasing $(-4,-2) \cup (0,2)$

(f) Now use all your answers to the questions to sketch a graph of the derivative function $f'(x)$ on the coordinate plane below.



2. Below is a graph of a derivative $g'(x)$. ~~Assume this is the entire graph of $g'(x)$~~. Use the graph to answer the following questions about the original function $g(x)$.



(a) On which interval or intervals is the original function $g(x)$ increasing?

$g'(x) > 0$ $(A, B) \cup (C, \infty)$

(b) On which interval or intervals is the original function $g(x)$ decreasing?

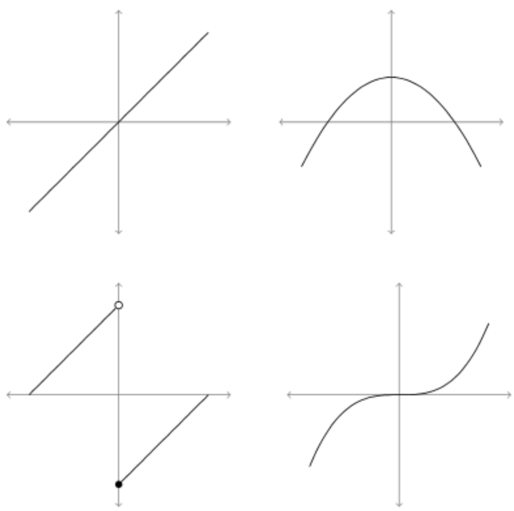
$g'(x) < 0$ $(-\infty, A) \cup (B, C)$

(c) Now suppose $g(0) = 0$. Is the function $g(x)$ ever positive? That is, is there any x so that $g(x) \gtrsim 0$? How do you know?

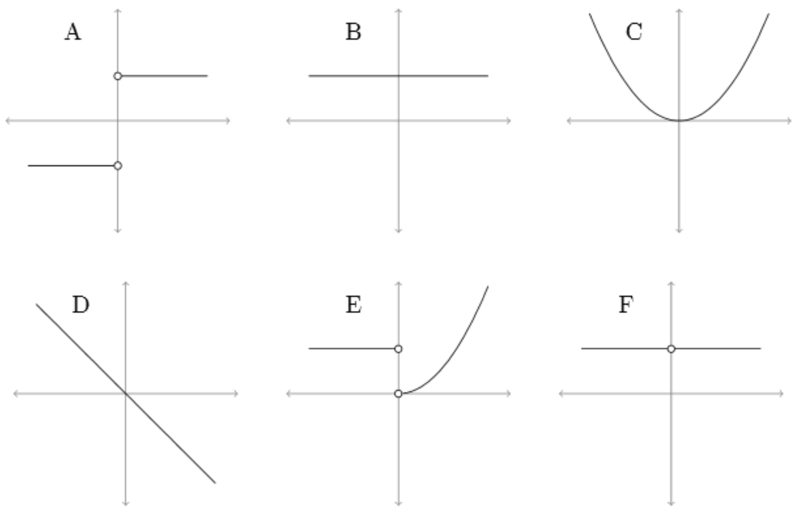
yes b/c the function is decreasing through (0,0)

3. Six graphs of functions are below, along with six graphs of derivatives. Match the graph of each function with the graph of its derivative.

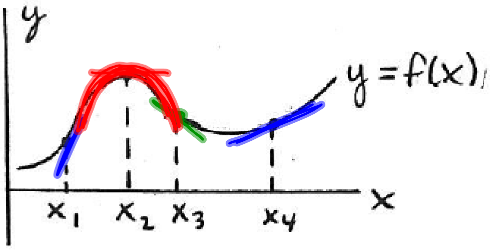
Original Functions:



Their derivatives:



Visual Estimates of Derivatives



$f''(x_2) < f(x_2)$

Using the graph of $y = f(x)$, fill in =, >, or <:

$f'(x_2) = 0$

$f'(x_4) > 0$

$f'(x_3) < 0$

$f'(x_1) > 0$

$f'(x_1) > f'(x_4)$

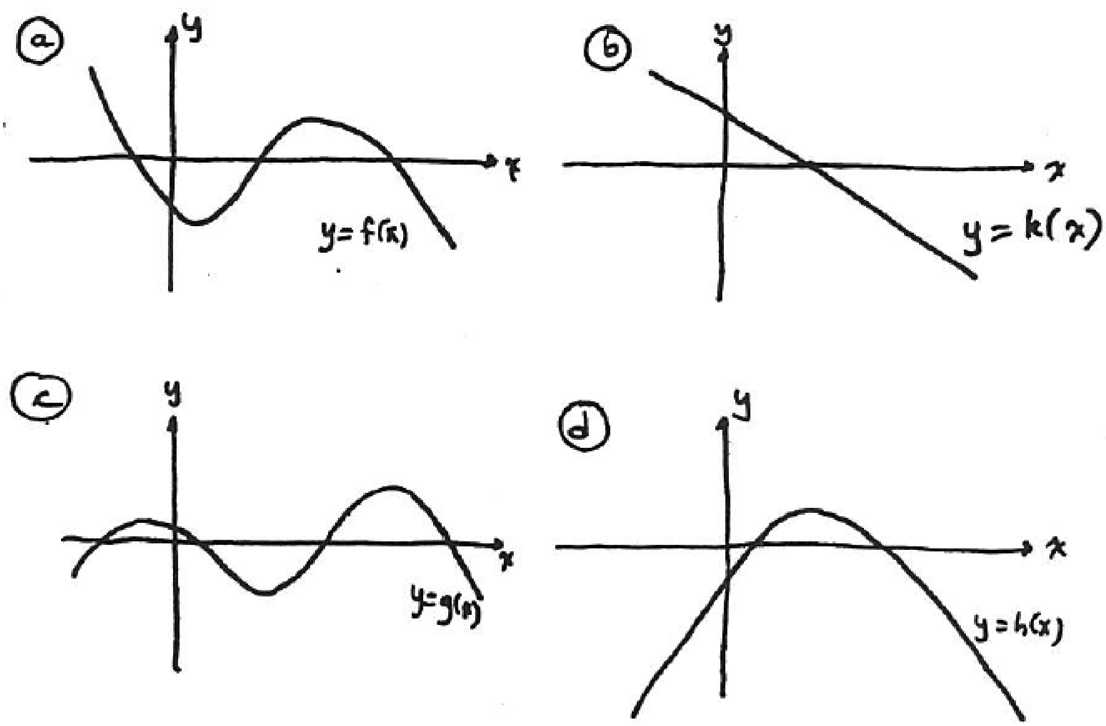
$f'(x_3) < f(x_3)$

$f'(x_2) < f'(x_1)$

$f'(x_2) > f'(x_3)$

$f'(x_3) < f'(x_4)$

GRAPHING THE DERIVATIVE



1. Let the graph in (d) be that of $y = h(x)$. Which of the others is the graph of $y = h'(x)$?
2. Which is the graph of $y = f'(x)$?
3. Which is the graph of $y = g'(x)$?
4. Which is the graph of $y = g''(x)$?
5. Which is the graph of $y = g'''(x)$?

HW Check:

$$f(x) = \frac{x^2-1}{x} \qquad f'(x) = \frac{x(2x)-(x^2-1)(1)}{x^2}$$

$$f'(x) = \frac{x^2+1}{x^2}$$

$$f''(x) = \frac{x^2(2x)-(x^2+1)(2x)}{x^4} = \frac{-2x}{x^4}$$

a) $f'(x)=0$

$x^2+1=0$

$x^2=-1$

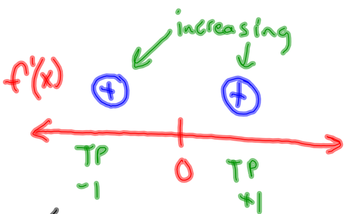
$x=\pm\sqrt{-1}$

imaginary

$f'(x)=$ undefined

$x^2=0$

$x=0$



$f(x)$ increasing: $(-\infty,0)\cup(0,\infty)$

$f(x)$ decreasing: Never

b) Top

$f''(x)=0$

$-2x=0$

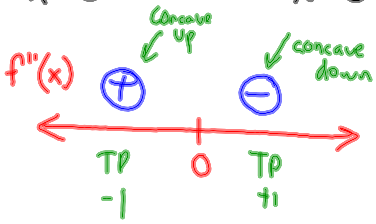
$x=0$

Bottom

$f''(x)=$ undefined

$x^4=0$

$x=0$



$f(x)$ is concave up: $(-\infty,0)$

$f(x)$ is concave down: $(0,\infty)$

c) It appears that there is an inflection point when $x=0$

Test 0 in the original $f(x)$

$$f(0) = \frac{0^2-1}{0} = \frac{-1}{0} = \text{undefined}$$

therefore no inflection pt.